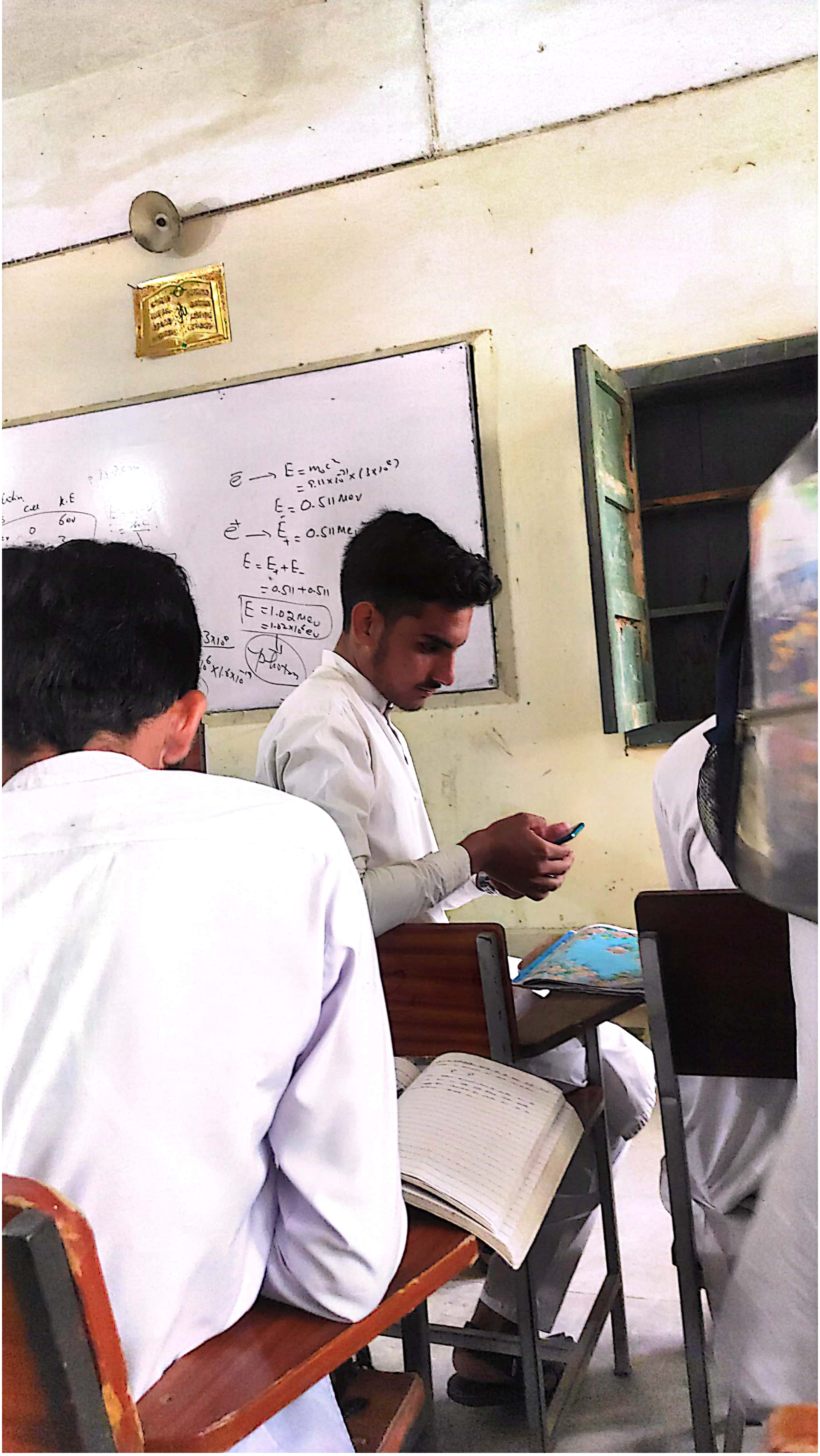
E = m₀c²
E = 0.511 MeV
E = E₊ + E₋
= 0.511 + 0.511
E = 1.02 MeV
Photon

Total Diff equation

$$df(x,y) = \frac{df}{dx}dx + \frac{\partial f}{\partial y}dy$$

$$f(x,y) = x^2 y$$

$$df(x,y) = y\,2x\,dx + x^2 dy$$

$$df(x,y) = 2xy\,dx + x^2 dy$$

$$(M(x,y)\,dx + N(x,y)\,dy)$$

Diff equation

$$M(x,y)\,dx + N(x,y)\,dy$$

$$= \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

exict equation,

$$M(x,y) = \frac{\partial f}{\partial x}$$

$$N(x,y) = \frac{\partial f}{\partial y}$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

When it's both equal then it's called exict equation

Integrating

$$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy$$

1) $(3x^2y+2)\,dx + (x^3+y)\,dy = 0$

Soln:

$(3x^2y+2)\,dx + (x^3+y)\,dy = 0$ — (1)

let

$M = 3x^2y+2$, $N = x^3+y$

$\frac{\partial M}{\partial y} = 3x^2(1)+0$, $\frac{\partial N}{\partial x} = 3x^2(1)$

$\frac{\partial M}{\partial y} = 3x^2 \longrightarrow$ (i) , $\frac{\partial N}{\partial x} = 3x^2$ — (ii)

As $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

so (1) is exact

so we can write

$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = C$

$\int(3x^2y+2)\,dx + \int y\,dy = C$

$\int 3x^2y + \int 2\,dx + \int y\,dy = C$

$\cancel{3}y \cdot \frac{x^3}{\cancel{3}} + 2x + \frac{y^2}{2} = C$

$x^3y + 2x + y^2/2 = C$

$2x^3y + 4x + y^2 = 2C$

$\boxed{C_1 = 2C}$

$\boxed{2x^3y + 4x + y^2 = C_1}$

Ans.

When we chang non exact equation to exact equation we multiply any function called integrating factor. (I.F).

2) $(2y\sin x\cos x + y^2\sin x)\,dx + (\sin^2 x - 2y\cos x)\,dy = 0$

Sol:

$M = (2y\sin x\cos x + y^2\sin x)$

$N = (\sin^2 x - 2y\cos x)$

For M

$\frac{\partial M}{\partial y} = 2\sin x\cos x(1)\sin x(2y)$

$\frac{\partial M}{\partial y} = 2\sin x\cos x + 2y\sin x$ — (i)

Now For N

$N = \sin^2 x - 2y\cos x$

$\frac{\partial N}{\partial x} = 2\sin x\cos x - 2y(-\sin x)$

$\frac{\partial N}{\partial x} = 2\sin x\cos x + 2y\sin x$ — (ii)

As $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so eq (1)

is exact diff equation

$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int (2y\sin x\cos x)\,dx + \int (0)\,dy = c$

$\int 2y\sin x\cos x\,dx + \int y^2\sin x + c_1 = c$

$2y\int \sin x\cos x\,dx + y^2\int \sin x + c_1 = c$

$2y\frac{(\sin x)^2}{2} + y^2(-\cos x) + c_1 = c$

$y\sin^2 x - y^2\cos x + c_1 = c$

$y\sin x - y^2\cos x + c_1 - c = 0$

$y\sin^2 x - y^2\cos^2 x + c_2 = 0$ — (iii)

Required value

$3(\sin(0)) - (3)^2\cos(0) + c_2 = 0$

$-9(1) + c_2 = 0$

$-9 = c_2$

$\boxed{c_2 = 9}$ put in (iii)

$y\sin^2 x - y^2\cos^2 x + 9 = 0$

Ans.

$x^2 \cdot 2x\,dx$

$\frac{x^{2+1}}{2+1} = \frac{x^3}{3}$

2) $(2xy + y - \tan y)\,dx + (x^2 - \tan^2 y + \sec^2 y)\,dy$

Sol:

$M = 2xy + y - \tan y$,

$\frac{\partial M}{\partial y} = 2x(1) + 1 - \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y$ — (i)

For $N = x^2 - \tan^2 y + \sec^2 y$

$\frac{\partial N}{\partial x} = -2x - \tan^2 y(1) + 0$

$= 2x - \tan^2 y$

$= 2x - (\sec^2 y - 1)$

$= 2x - \sec^2 y + 1$

$= 2x + 1 - \sec^2 y$ — (ii)

So $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(1) is exact diff eq

$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int (2xy + y - \tan y)\,dx + \int \sec^2 y\,dy = c$

$\int 2xy\,dx + \int y\,dx - \int \tan y\,dx + \int \sec^2 y\,dy = c$

$2y \int x\,dx + y \int dx - \tan y \int dx$

3) $(1+\ln xy)dx+(1+x/y)\,dy=0$

Sol.

$\Rightarrow (1+\ln xy)dx+(1+x/y)\,dy=0$ — (1)

Let

$M=1+\ln(xy)$, $N=1+x/y$

$\frac{\partial M}{\partial y}=0+\frac{1}{xy}\frac{d}{dx}(xy)$

$\frac{\partial M}{\partial y}=\frac{1}{x}\,y\cdot x\,(1)$

$\frac{\partial M}{\partial y}=1/y$ — (i)

For $N=1+x/y$

$\frac{\partial N}{\partial x}=0+1/y$

$\frac{\partial N}{\partial x}=1/y$ — (ii)

So (i) is exact diff eq

$\int M\,dx+\int(\text{term of } N \text{ free from } x)\,dy=$

$\int(1+\ln xy)\,dx+\int 1\,dy=C$

$$\int dx + \int \ln xy \, dx + \int dy = c$$

$$x + \int \ln(xy) \cdot 1 \, dx + y = c$$

$$x + \ln xy \int 1 \, dx - \int \frac{d}{dx} \ln xy \int (1 dx) + y = c$$

$$x + \ln xy - x \int \frac{1}{xy} \frac{d}{dx}(x \cdot y) - x + y = c$$

$$x + \ln xy \, (x) - \int \frac{1}{\cancel{x}y} \, y \, (\cancel{x}) \, dx + y = c$$

$$x + x \ln xy - \int dx$$

$$= x + x \ln xy - \int dx + y = c$$

proved

yha ak question nahi lika

Non exact diff eq

An equation

$M(x,y)\,dx + N(x,y)\,dy = 0$ — (1)

(1) is said to be non exact diff equation if $M_y \neq N_x$

if non exact equation solve the we multiply any function $\mu(x,y)$ called integratin function

$\mu(x,y)\,M(x,y)\,dx + N(x,y)\,dy = 0$

Integrating factor:—

if diff equation

$M(x,y)\,dx + N(x,y)\,dy = 0$ — (1)

is non exact equation.

Rule (1)

$\frac{M_y - N_x}{N}$ = function in term of x only

$\left(x^2,\ x^2+1,\ \frac{x^2+1}{x^2-1} \right)$

$\int (x^2)\,dx$

I.F $= e$

Rule (2)

$\frac{N_x - M_y}{M}$ function in term of y only ( )

$\int (\ ) \, dy$

$I.F = e$

Rule ③

$M(x,y)\,dx + N(x,y)\,dy = 0$ is in homogeneous form.

Then $xM + yN \neq 0$

$$I.F = \frac{1}{xM + yN}$$

Rule ④ :-

$xM - yN \neq 0$

$$I.F = \frac{1}{xM - yN}$$

Solve the following eq by find integral factor (I.F)

Q1) $y\,dx + (x^2 y - x)\,dy = 0$ — ①

Sol:-

$M = y \quad , \quad N = x^2 y - x$

$M_y = 1 \qquad N_x = y\,2x - 1$

$M_y \neq N_x$

so eq ① is non exact eq.

Rule (1)

$$\frac{My - Nx}{N}$$

$$\frac{1 - (2xy - 1)}{x^2 y - x}$$

$$= \frac{1 - 2xy + 1}{x^2 y - x}$$

$$= \frac{2 - 2xy}{x^2 y - x}$$

$$= \frac{2(1 - xy)}{x(xy - 1)}$$

$$= \frac{-2\cancel{(xy-1)}}{x\cancel{(xy-1)}}$$

$$\frac{My - Nx}{N} = -2/x$$

Rule (1) is satisfied.

so $\int (-2/x)\, dx$

$I.F = e$

$$I.F = e^{-2\int \frac{1}{x} dx}$$

$$= e^{-2 \ln x}$$

$$= \cancel{e^{\ln x^{-2}}} = x^{-2}$$

$$I.F = \frac{1}{x^2}$$

Multiply eq by $1/x$.

① $\Rightarrow \quad y\,dx + (x^2y - x)\,dy = 0$

$\frac{1}{x^2}\, y\,dx + \frac{1}{x^2}(x^2y - x)\,dy = 0$

$\frac{y}{x^2}\,dx + \left(y - \frac{1}{x}\right)dy = 0$

is an exact diff equation

$M = \frac{y}{x^2}$, $N = y - \frac{1}{x}$

we know that

$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = c$

$\int \frac{y}{x^2}\,dx + \int y\,dy = C$

$y\int \frac{1}{x^2}\,dx + \int y\,dy = C$

$y\int x^{-2}\,dx + \int y\,dy = C$

$y\,\frac{x^{-2+1}}{-2+1} + \frac{y^2}{2} + C_1 = C$

$y\,\frac{x^{-1}}{-1} + \frac{y^2}{2} = C - C_1$

$\frac{-y}{x} + \frac{y^2}{2} = C_2$

Required Solution

$(xy^2 + y)\,dx - x\,dy = 0$ (i)

Solution:-

$M = xy^2 + y$

$M_y = 2yx + 1$

$M_y \neq N_x$

So equation 2 is non exact diff equation

Rule No 1

$$\frac{M_y - N_x}{N}$$

$$= \frac{2yx + 1 + 1}{-x}$$

$$= \frac{2yx + 2}{-x} = \frac{2(yx + 1)}{-x} \text{ (fail)}$$

Rule 2

$$\frac{N_x - M_y}{M}$$

$$\frac{-1 - (2yx + 1)}{xy^2 + y}$$

$$= \frac{-1 - 2yx - 1}{xy^2 + y}$$

$$\frac{-2 - 2yx}{xy^2 + y} = \frac{-2(\cancel{1 + yx})}{y(\cancel{xy + 1})}$$

$$= \frac{-2}{y}$$

So

$$I.F = e^{\int \left(\frac{-2}{y}\right) dy} = I.F = e$$

$$= e^{-2 \int \frac{1}{y} dy}$$

$$= e^{-2 \ln y} = e^{\ln y^{-2}}$$

$$I.F = y^{-2} = \frac{1}{y^2}$$

Xing eq (1) by $1/y^2$

$$1/y^2 \left((xy^2+y)\,dx\right) - 1/y^2 (x)\,dy = 0$$

$$\left(x\frac{\cancel{y^2}}{\cancel{y^2}} + \frac{\cancel{y}}{\cancel{y}^2}\right) dx - \frac{x}{y^2}\,dy = 0$$

$\left(x + 1/y\right) dx - \frac{x}{y^2}\,dy = 0$ is exact diff equation

$M = x + 1/y \qquad N = \frac{-x}{y^2}$

so we know that

$$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$$

$$\int \left(x + 1/y\right) dx + \int 0 \cdot dy = c$$

$$\int x\,dx + \int \frac{1}{y}\,dx = c$$

$$\frac{x^2}{2} + \frac{1}{y} \int dx = c$$

$$\frac{x^2}{2} + \frac{1}{y} \int dx + c_1 = c$$

$$\frac{x^2}{2} + \frac{x}{y} = c - c_1$$

$$\frac{x^2}{2} + \frac{x}{y} = c$$

Home work.

(3) $(x^2 + x - y)\,dx + x\,dy = 0$

(4) $y(2xy + e^x)\,dx - e^x\,dy = 0$

(5) $(3y + 4xy^2)\,dx + (2x + 3x^2y)\,dy = 0$

Linear Diff eq of Order:—

A diff. eq is said to be linear diff eq of order 1 with dependent variable y and independent variable x

$$\frac{dy}{dx} + p(x)y = Q(x)$$

$$\frac{dr}{d\theta} + p(\theta)r = Q(\theta)$$

$$\frac{dx}{dy} + p(y)x = Q(y)$$

$$\frac{dy}{dx} + \left(\frac{x^2+1}{x^2-1}\right)y = (x^4)$$

if we can write eq as

$$\frac{dy}{dx} + p(x)y = Q(x) \quad \text{--- (1)}$$

where $p(x) = Q(x)$

function of x.

Solving of linear eq. (1)

$$p(x) = e^{\int p(x)\,dx} = \boxed{}$$

Multiplying with (1)

LHS = Total diff equation.

$$\frac{d}{dx}(f+g)$$

$$f\frac{d}{dx}f + g\frac{dg}{dx}$$

Solve the following eq.

$$\frac{dy}{dx} + \frac{4y}{x-1}y = \frac{x+1}{(x-1)^3} \quad \text{--- (1)}$$

here $p(x) = \frac{4}{x-1}$

$$I.F = e^{\int p(x)\,dx}$$

$$I.F = e^{\int \frac{4}{x-1}dx}$$

$$I.F = e^{4\int \frac{1}{x-1}dx}$$

$$I.F = e^{4\int \frac{1}{x-1} dx}$$

$$I.F = e^{4\ln(x-1)}$$

$$I.F = e^{\ln(x-1)^4}$$

$$I.F = (x-1)^4$$

$\times$ing with eq (i)

$$(x-1)^4 \frac{dy}{dx} + (x-1)^4 \frac{4}{x-1} y = \frac{x+1}{(x-1)^3} (x-1)^4$$

$$\frac{d}{dx}\left((x-1)^4 y\right) = x^2-1$$

$$d\left((x-1)^4 y\right) = (x^2-1)\, dx$$

Integrating

$$\int d(x-1)^4 y = \int (x^2-1)\, dx$$

$$(x-1)^4 y = \int x^2 dx - \int dx + c$$

$$(x-1)^4 y = \frac{x^{2+1}}{2+1} - x + c$$

$$(x-1)^4 y = \frac{x^3}{3} - x + c$$

$$y = \frac{x^3}{3(x-1)^4} - \frac{x}{(x-1)^4} + \frac{c}{(x-1)^4}$$

Required general solution.

(2) $\frac{dy}{dx} + \frac{(2x+1)}{x} y = e^{-2x}$ — (1)

Linear diff eq in y.

$p(x) = \frac{2x+1}{x}$

$I.F = e^{\int p(x)dx}$

$I.F = e^{\int \frac{2x+1}{x} dx}$

$= e^{\int \left(\frac{2\cancel{x}}{\cancel{x}} + 1/x \right) dx}$

$e^{\int 2 dx \int 1/x \, dx}$

$e^{\int 2 dx} = e^{\ln x}$

$e^{2x} \cdot e^{\ln x}$

$e^{2x + \ln x}$

$e^{2x} \cdot e^{\ln x}$

$\boxed{I.F = e^{2x} \cdot x}$

Xing eq (1)

$xe^{2x} \frac{dy}{dx} + xe^{2x} \frac{(2x+1)}{x} y = xe^{2x} \cdot e^{-2x}$

$xe^{2x} \frac{dy}{dx} + e^{2x}(2x+1) y = x$

$$\frac{d}{dx}\left(xe^{2x}y\right) = x$$

Integrating

$$xe^{2x}y = \int x\,dx + c$$

$$xe^{2x}y = \frac{x^2}{2} + c$$

$$\boxed{y = \frac{x^2}{2xe^{2x}} + \frac{c}{2xe^{2x}}}$$

$$\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x^2}{\ln x}$$

Sol:

$$\frac{dy}{dx} + \frac{1}{x\ln x}y = \frac{3x^2}{\ln x} \quad \text{— (1)}$$

linear diff eq in y

so

$$p(x) \quad \frac{1}{x\ln x}$$

$$\boxed{\int \frac{2x}{x^2}\,dx \quad \ln x^2}$$

$$I.F = e^{\int p(x)\,dx}$$

$$= e^{\int \frac{1}{x\ln x}\,dx}$$

$$= e^{\int \frac{\frac{1}{x}}{\ln x}\,dx}$$

$$= \cancel{e}^{\cancel{\ln}\ln x} \qquad = \ln x$$

Mul eq (1)

$$\ln x\frac{dy}{dx} + \cancel{\ln x}\frac{1}{x\cancel{\ln x}}y = \frac{3x^2}{\cancel{\ln x}}\cancel{\ln x}$$

$$= \ln x \frac{dy}{dx} + \cancel{\ln x} \frac{1}{x \cancel{\ln x}} y = 3x^2$$

$$= \ln x \frac{dy}{dx} + \frac{1}{x} y = 3x^2$$

$$\frac{d}{dx} (\ln x . y) = 3x^2$$

$\frac{d}{dx}$ ~~d~~ integrating

$$\ln x . y = \int 3x^2 + c$$

$$\ln x . y = \frac{3x^3}{3} + c$$

$$\ln x . y = x^3 + c$$

$$y = \frac{x^3}{\ln x} + \frac{c}{\ln x}$$

4) $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$

Sol, $p(x) = 3$ $\quad$ I.F $e^{\int 3 \, dx}$

$$I.F = e^{3x}$$

5) $\cos^3 x \dfrac{dy}{dx} + y\cos x = \sin x$

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \frac{1}{\cos^2 x}\, y = \frac{\sin x}{\cos^3 x} \quad (1)$$

Linear diff eq in y

$$P(x) = \frac{1}{\cos^2 x}$$

$$I.F = e^{\int \frac{1}{\cos^2 x} dx}$$

$$I.F = e^{\int \sec^2 x \, dx}$$

$$I.F = e^{\tan x}$$

Multiply eq (1)

$$e^{\tan x}\frac{dy}{dx} + e^{\tan x}\frac{1}{\cos^2 x} y = e^{\tan x}\frac{\sin x}{\cos^3 x}$$

$$\frac{d}{dx}\left[e^{\tan x} \cdot y\right] = e^{\tan}\frac{\sin x}{\cos^3 x}$$

integrating both side

$$e^{\tan} \cdot y = \int e^{\tan x} \cdot \frac{\sin x}{\cos^3 x} dx + c$$

$$= \int e^{\tan x} \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^2 x} dx + c$$

$$e^{\tan x} y = \int e^{\tan x} \cdot \tan x \cdot \sec^2 x \, dx + c \rightarrow (A)$$

$$\frac{dy}{dx} = \frac{1}{e^{y-x}}$$

Sol:- $\frac{dx}{dy} = e^{y} - x$

$$\frac{dx}{dy} + x = e^{y} \quad \text{--- (1)}$$

Linear diff eq in x

$$p(y) = 1$$

$$I.F = e^{\int p(y)\,dy}$$

$$I.F = e^{\int 1\,dy}$$

$$IF = e^{y}$$

Multiply eq (1)

$$e^{y}\frac{dx}{dy} + e^{y}x = e^{y}e^{y}$$

$$\frac{d}{dy}(e^{y} \cdot x) = e^{2y}$$

integrating.

$$e^{y}x = \int e^{2y} + c$$

$$e^{y}x = \frac{e^{2y}}{2} + c$$

$$x = \frac{e^{2y}}{2e^{y}} + \frac{c}{e^{y}}$$

$$x = \frac{e^{2y-y}}{2} + \frac{c}{e^{y}} \Rightarrow x = \frac{e^{y}}{2} + \frac{c}{e^{y}}$$

Find the solution of the following Barnaulli's equation.

(1) $\frac{dy}{dx} + \frac{x}{1-x^2} y = x y^{1/2}$ —— (1)

Solve:

dividing b/s by $y^{1/2}$

$y^{-1/2} \cdot \frac{dy}{dx} + \frac{x}{1-x^2} y^{1-1/2} = x$

$y^{-1/2} \frac{dy}{dx} + \frac{x}{1-x^2} y^{1/2} = x$ —— (2)

let $v = y^{1/2}$ —— (i)

$\frac{dv}{dx} = \frac{1}{2} y^{1/2 - 1} \frac{dy}{dx}$

$\frac{dv}{dx} = \frac{1}{2} y^{-1/2} \frac{dy}{dx}$

$\Rightarrow 2\frac{dv}{dx} = y^{-1/2} \frac{dy}{dx} \longrightarrow$ (ii)

put (i) and (ii) in (2)

$2\frac{dv}{dx} + \frac{x}{1-x^2} v = x$

$\Rightarrow \frac{dv}{dx} + \frac{x}{2(1-x^2)} v = \frac{x}{2}$ —— (3)

Linear diff in v

let $p(x) = \frac{x}{2(1-x^2)}$

$$I.F = e^{\int p(x)\,dx}$$

$$I.F = e^{\frac{1}{2}\int \frac{x}{1-x^2}\,dx}$$

Dividing ② on b/s

$$I.f = e^{-\frac{1}{4}\int \frac{-2x}{1-x^2}\,dx}$$

$$I.f = e^{-\frac{1}{4}\ln(1-x^2)}$$

$$\cancel{e}^{\ln(1-x^2)^{-1/4}}$$

$$= (1-x^2)^{-1/4}$$

$$I.f = \frac{1}{(1-x^2)^{1/4}}$$

All with eq ③

$$\Rightarrow \frac{1}{(1-x^2)^{1/4}}\frac{dv}{dx} + \frac{xv}{2(1-x^2)^{5/4}} = \frac{x}{2(1-x^2)^{1/4}}$$

$$\frac{d}{dx}\left[\frac{1}{(1-x^2)^{1/4}} \cdot v\right] = \frac{x}{2(1-x^2)^{1/4}}$$

$$\frac{1}{(1-x^2)^{1/4}} \cdot v = \frac{1}{2}\int \frac{x}{(1-x^2)^{1/4}}\,dx$$

$$= \frac{1}{2}\int (1-x^2)^{-1/4}\, x$$

$= \frac{1}{2}\int (1-x^2)^{-1/4} . x$

$= \frac{1}{2}\int (1-x^2)^{-1/4} . \left(\frac{-2x}{-2}\right) dx$

$= -\frac{1}{4}\int (1-x^2)^{-1/4} (-2x)\, dx$

$= \frac{-1}{4} (1-x^2)^{-1/4+1} + C$

$= -\frac{1}{4} (1-x^2)^{3/4} + C$

$\frac{1}{(1-x^2)} v = \frac{-1}{\cancel{4}} \frac{(1-x^2)^{3/4}}{3/\cancel{4}} + C$

replace
$v = y^{1/2}$

$\frac{1}{(1-x^2)^{1/4}} y^{1/2} = -\frac{(1-x^2)^{3/4}}{3} + C$

13) $x\frac{dy}{dx} + y = y^2 \ln x$

Soln

$\frac{x}{y^2} \frac{dy}{dx} + \frac{y}{y^2} = \ln x$

$\Rightarrow y^{-2} x \frac{dy}{dx} + y^{-1} = \ln x$ — 1 let $v = y^{-1}$

$$\frac{dv}{dx} = -y^{-1-1} \, dy/dx$$

$$\frac{dv}{dx} = -y^{-2}\frac{dy}{dx} \quad —— (iii)$$

so

$$① \Rightarrow x\frac{dv}{dx} + v = \ln x$$

$$\frac{dv}{dx} - \frac{v}{x} = -\frac{\ln x}{x} \rightarrow ②$$

$$p(x) = -1/x$$

so Integrating factor

$$e^{\int -1/x \, dx}$$

$$e^{-\ln x}$$

$$e^{\ln x^{-1}}$$

$$x^{-1}$$

$$I.F = 1/x$$

$\times$ ing with ②

$$\frac{1}{x}\cdot\frac{dv}{dx} - \frac{1}{x}\cdot\frac{v}{x} = -\frac{\ln x}{x}\cdot\frac{1}{x}$$

$$\left(\frac{1}{x}\frac{dv}{dx} - \frac{v}{x^2}\right) = -\frac{\ln x}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{x}\cdot v\right) = -\frac{\ln x}{x^2}$$

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